Closed Integral Form Expansion of Raman Equation for Efficient Gain Optimization Process

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***Abstract*—In this study, we introduce a closed integral form of coupled Raman equation set, which enables us to reduce the complex multipump Raman gain design problem into a series of matrix calculations. With simultaneous implementation of signal stimulated Raman scattering effect and Raman integration in single iteration step, fast convergence with excellent accuracy has been achieved.**

***Index Terms*—Iterative method, modeling, optimization method, Raman fiber amplifiers (RFAs), wavelength-division multiplexing.**

I. Introduction

Out of many distinctive advantages that the Raman fiber amplifier (RFA) provides, the flexibility in gain engineering can be considered to be one of the key advantages that other types of amplifier cannot provide. However, most common techniques for RFA's modeling based on the coupled ordinary differential equation (ODE) require exhaustive computational efforts, to achieve reasonable accuracies in the prediction of RFAs' performances. Even though it is now possible to get a significant reduction in the computation time (up to three orders) through efficient formulations of Raman equation [1], still there exists additional difficulty in the gain profile design—as the interactions of pump waves inside the fiber make it difficult to predict the required pump power sets for the target gain profile [2]. To trace back the pump powers required for the target design, one has to iteratively adjust pump input powers by using either 1) the amount of gain error between the signal gain and target spectrum—equivalent to the shooting method in one form or another—with different tracking algorithms [3], or 2) rescaled injection pump power integrals obtained from the numerical integration of coupled differential equations [4].

Still, all of these aforementioned approaches require a time-consuming process of repetitive numerical integrations for coupled ODE, including all of the involved signal–pump waves.

In this work, we propose a novel algorithm based on newly derived Raman *integral equations* for the rapid optimization of multiwavelength gain profile design. By approaching the Raman gain design problem along an iteration axis with the proposed closed form equations, highly accurate estimation of pump powers and fast construction of gain profiles can be achieved with much reduced efforts, compared to prior arts.

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II. Formulation

The coupled nonlinear equations describing the Raman process can be expressed as follows:

$$\pm\frac{dP_i}{dz} = -\alpha_i P_i + \sum_{j=1}^{M+N} g_{ji} P_j P_i \tag{1}$$

where P_i is the power at ith wavelength, α_i is the attenuation coefficient, g_{ji} is the Raman gain coefficient, and M and N are the number of pump and signal waves, respectively. After dividing (1) with P_i and integrating over z, we get

$$\int_0^z \left(\frac{1}{P_i}\frac{dP_i}{dz}\right) dz = \mp\,\alpha_i z \pm \left(\sum_{j=1}^{M+N} g_{ji}\int_0^z P_j\,dz\right)$$
$$P_i(z) = P_i(0)\exp\left[\mp\alpha_i z \pm \left(\sum_{j=1}^{M+N} g_{ji}\int_0^z P_j\,dz\right)\right]. \tag{2}$$

With additional integration process, we get the integral form of Raman wave equation as expressed below

$$\frac{\int_0^z P_i(z)\,dz}{P_i(0)} = \int_0^z \exp\left[\mp\alpha_i z \pm \left(\sum_{j=1}^{M+N} g_{ji}\int_0^z P_j\,dz\right)\right] dz. \tag{3}$$

Utilizing the definition of $L_{\text{eff\_}i}(z) = \int_0^z P_i(z)/P_i(0)dz$, then (3) can be put into a following closed integral form:

$$L_{\text{eff\_}i}(z) = \int_0^z \exp\left[\mp\alpha_i z \pm \left(\sum_{j=1}^{M+N} g_{ji}P_j(0)L_{\text{eff\_}j}(z)\right)\right] dz. \tag{4}$$

To solve above closed form of equation set for effective length $L_{\text{eff\_}i}s$, we apply Picard's iteration method to get

$$L_{\text{eff\_}i}^{(n)}(z) = \int_0^z \exp\left[\mp\alpha_i z \pm \left(\sum_{j=1}^{M+N} g_{ji}P_j^{(n-1)}(0)L_{\text{eff\_}j}^{(n-1)}(z)\right)\right] dz \tag{5a}$$

$$\text{where}\ \begin{cases} g_{ji} = \frac{g_R(v_j - v_i)}{2A_{\text{eff}}}\left(\frac{\lambda_R}{\lambda_{Pj}}\right)^k, & (\text{when } j \le i) \\ g_{ji} = -\left(\frac{v_i}{v_j}\right)\frac{g_R(v_i - v_j)}{2A_{\text{eff}}}\left(\frac{\lambda_R}{\lambda_{Pj}}\right)^k, & (\text{when } j > i) \end{cases}$$

$$L_{\text{eff\_}Ii}^{(n)}(z_{\_K}) = \sum_{z'_{\_K}=0}^{z_{\_K}} \exp\left[\mp\alpha_i z \pm \left(\sum_{j=1}^{M+N} g_{ji}P_j^{(n-1)}(0)L_{\text{eff\_}Ij}^{(n-1)}(z'_{\_K})\right)\right] \cdot \Delta z. \tag{5b}$$

Here, λ_R is the wavelength of reference gain and k is a frequency scale factor. Under this formalism, the complex coupled differential equation problem for RFA now has been reduced to a simple iterative calculation procedure searching for the effective length $L_{\text{eff}}(z)$. For the numerical integration process, we defined $L^{(n)}_{\text{eff\_}Ij}(z_{\_K})$ as the discretized value for effective length at $z_{\_K}$ with step Δz. It is worth noting that 5(a) can now be expressed in terms of a matrix format 5(b) between $L^{(n)}_{\text{eff\_}Ii}(z_{\_K})$ and $L^{(n-1)}_{\text{eff\_}Ij}(z_{\_K})$. The scaled gain coefficients $(g_R/2A_{\text{eff}})$ at different wavelengths also can be easily obtained from nonintrusive measurement with wavelength scaling technique [5].

III. Algorithm

Utilizing the inhomogeneous property of Raman scattering process, we can rewrite a common expression for signal Raman ON–OFF gain [6] into a matrix form (7)

$$\begin{aligned} G_{\text{dB\_}t} &= \sum_{j=1}^{M+N} G_{\text{dB\_}j} \\ &= \sum_{j=1}^{M+N} \left(10\log(e)\frac{g_R}{2A_{\text{eff}}}\left(\frac{\lambda_R}{\lambda_j}\right)^k \right)\cdot P_j(0)\cdot L_{\text{eff\_}j} \\ &= \widetilde{T}_P\cdot\vec{P}_P + \widetilde{T}_S\cdot\vec{P}_S \end{aligned} \tag{6}$$

where

$$\begin{aligned} \widetilde{T}_{I=P,S} &= \begin{bmatrix} G^{(1,1)}_{RI}L_{\text{eff\_}I1} & \cdots & G^{(1,K)}_{RI}L_{\text{eff\_}IK} \\ \vdots & \vdots & \vdots \\ G^{(N,1)}_{RI}L_{\text{eff\_}I1} & \cdots & G^{(N,K)}_{RI}L_{\text{eff\_}IK} \end{bmatrix}_{K=M,N} \\ \vec{P}_{I=P,S} &= \begin{bmatrix} P_{I1}(0) \\ \vdots \\ P_{IK}(0) \end{bmatrix}_{K=M,N} \\ G^{(i,j)}_{RI} &= 10\log(e)\frac{g_R(\lambda_i\lambda_j)}{2A_{\text{eff}}}\left(\frac{\lambda_R}{\lambda_j}\right)^k \end{aligned} \tag{7}$$

with index $I = P, S$ for pump and signal waves, respectively.

From this, we can begin the gain design procedure for multipumped RFA by constructing $G_{\text{dB\_}t}$ under a given set of constraints (bandwidth, flatness, available pump numbers, etc.) to determine the best achievable *ideal Raman gain* profile, from the scaled sum of single gain profiles $G_{\text{dB\_}j}$ contributed by each wave. Through simple iteration/fitting procedures between $G_{\text{dB\_}t}$ and *ideal target gain* profile, the strength of gain contributions $G_{\text{dB\_}j}$ from each wavelength (or the product of power and effective length $P_j(0)\cdot L_{\text{eff\_}j}$) then can be determined. Ideally, the required injection pump power $P_{Pj}(0)$ also can be obtained using the information on effective length $L_{\text{eff\_}j}$ (or equivalently, pump integrals in [4]) at each wavelength. To practically acquire the solution set for a given problem stated above, we follow:

Step I : At first, considering the nature of adiabatic expansion where we treat $g_{ji}\cdot P_j(0)\cdot L_{\text{eff\_}j}$ as a perturbation parameter, $L^{(0)}_{\text{eff\_}Ij}$ should be equal to $(1 - \exp(-\alpha_j L))/\alpha_j$. With $L^{(0)}_{\text{eff\_}Ij}$, we then can get initial estimations for transfer matrices $T^{(0)}_P$ and $T^{(0)}_S$. In good approximation, $T^{(0)}_S$ also can be ignored in this first step of iteration procedure, since $g_R(\nu_{si} - \nu_{sj}) \ll g_R(\nu_{pi} - \nu_{sj})$, and $P_s \ll P_p$. This last step enables us to pseudoinvert the transfer matrix in **(7)**, to give $P^{(0)}_{Pj}(0) = (T^{(0)}_P)^{-1}\cdot G_{\text{dB\_}t}$. It is worth noting that the effective length evolution (over z) data $L^{(0)}_{\text{eff\_}Ij}(z) = (1 - \exp(-\alpha_j z))/\alpha_j$ needs to be generated in this step, to be used in the next stage of iteration process.

Step II : Substituting the $P^{(0)}_{Ij}(0)$ and $L^{(0)}_{\text{eff\_}Ij}(z)$ into **5(a)**, we get the updated $L^{(1)}_{\text{eff\_}Ij}(z)$ and $L^{(1)}_{\text{eff\_}Ij}(L) = L^{(1)}_{\text{eff\_}Ij}$ for updated matrices $T^{(1)}_P$ and $T^{(1)}_S$. Then we get $P^{(1)}_{Pj}(0)$ from $(T^{(1)}_P)^{-1}\cdot(G_{\text{dB\_}t} - T^{(1)}_S P_S)$, where $P_S = P^{(n)}_{Sj}(0)$ is the fixed boundary value inversely determined from the *ideal Raman gain* $G_{\text{dB\_}t}$ and predetermined input signal power.

Step III : Reiterating *Step II* for higher orders, we acquire the final solution set.

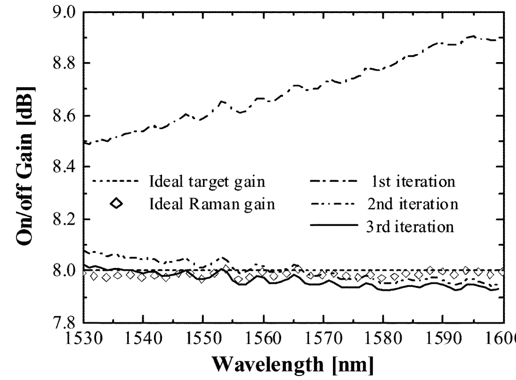

Fig. 1. Gain profiles at various stages of optimization procedure.

IV. Application Examples

To verify our algorithm, we carried out the design of a wide-band RFA, following formerly described procedures. For the first example, we set $G_{\text{dB\_}t}$ (*ideal Raman* ON–OFF *gain profile*) to be 8 ± 0.04 dB over 70 nm ($1530 \sim 1600$ nm) spectral range. Considering practical restrictions in the pump wavelength-division-multiplexing wavelength allocation, 16 equally spaced pump waves were used between $1420 \sim 1495$ nm, with 5-nm spacing. Also assumed were 70 signal waves at -3-dBm channel input power (1-nm spacing, $1530 \sim 1600$ nm). To test the robustness of our algorithm, we employed 37 km of fiber with large Raman gain coefficients (dispersion-shifted fiber instead of standard single-mode fiber).

Fig. 1 shows the ideal target gain (horizontal solid line) and ideal Raman gain $G_{\text{dB\_}t}$ obtained from the summation of individual pump contributions. Also shown in Fig. 1 are gain profiles generated from the full Raman differential equations, using the pump powers obtained after our first, second, and third iteration processes. As can be seen from the figure, the convergence error from the iteration procedure becomes almost negligible after the first two iteration steps. The observed absolute

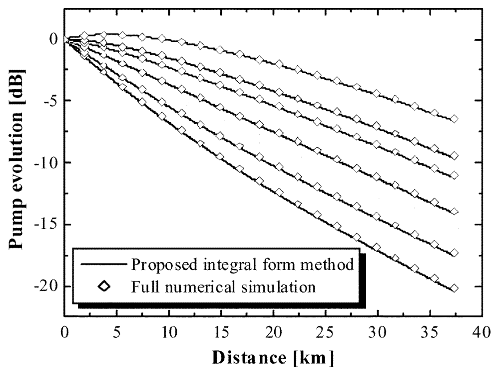

Fig. 2. Relative pump power along the fiber after fourth iteration (1420 $\sim$ 1480 nm with 10-nm step and 1495 nm, from bottom to top).

error between ideal target gain and the iteratively obtained gain profile after three iteration steps was smaller than 0.08 dB over the whole 70-nm gain spectral range. It is worth noting here that the level of gain accuracy could be even more improved with additional iteration steps by monitoring the convergence of $L^{(1)}_{\text{eff\_}Ij}(z)$ or $\langle \Delta G \rangle_{\text{AVG}}$, but considering most of the practical applications under experimental error, four iteration step was sufficient (for $\langle \Delta G \rangle_{\text{AVG}}$ improvement <0.005 dB).

Shown in Fig. 2 are the normalized pump evolution profiles along the fiber, after the four iteration steps. The difference $L^{(3)}_{\text{eff\_}Ij}(z)$ and $L^{(4)}_{\text{eff\_}Ij}(z)$ was smaller than 0.2% for all the wavelengths of interest. It is worth mentioning, with $L_{\text{eff\_}Sj}(z)$, other parameters of interest in RFA (such as amplified spontaneous emission, double Rayleigh scattering, etc.) also can be easily calculated from well-known equations [7]. The required computation time to obtain this result after four steps of iteration was less than 15 s with a conventional personal computer (2.0-GHz central processing unit clock), including all the time required for the initial generation of the ideal Raman gain profile (for comparison, the direct integration of Raman amplifier differential equation usually takes a few minutes even for a single scan).

We attribute this unprecedented level of convergence speed to 1) removing the use of time-consuming coupled ODE in the optimization process; 2) transformation of coupled Raman equations into closed integral equation form, which enable the *solution to be found along the iteration axis* not the fiber propagation axis; 3) matrix formulation for both of the signal stimulated Raman scattering (SRS) effect and effective length iteration equation—enabling fast numerical process in the software; and 4) simultaneous application of signal SRS effect and pump integral update in every iteration step.

Illustrated in Fig. 3 are design examples with this algorithm for various shapes of ideal target gain profiles (under identical setting as that of Fig. 1). It is also worth noting here that for all practical application purposes, we did not have to use all the signal channels, but it was sufficient to simulate the effect of signal SRS tilt and signal-pump depletion with wavelength-averaged tone (as long as the number of saturating tone is larger than the number of pump waves). For example, the gain difference between the 16-tone signal and 80 full channels loading case was smaller than 0.01 dB.

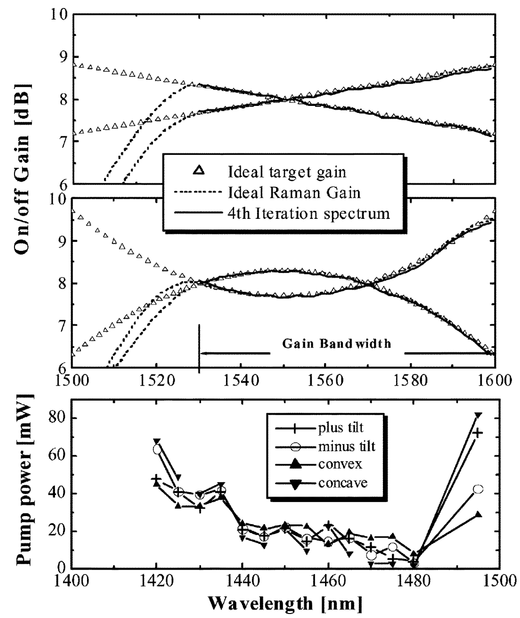

Fig. 3. Designed gain curves and corresponding pump powers under different target gain spectrum.

V. Conclusion

We propose a highly efficient algorithm for RFA gain profile design, based on an iterative matrix multiplication procedure enabled by newly suggested closed form of Raman integral equation. With simultaneous implementation of signal SRS effect and Raman integration in single iteration step, excellent accuracy (<0.08 dB) in the gain design with small number of iteration steps ($\leq$4 steps) has been achieved for most of the complex target gain profiles.

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