Adiabatic, closed-form approach to the highly efficient analysis of a fiber Raman amplifier problem

Lark Kwon Choi, Pilhan Kim, Jaehyoung Park, Jonghan Park, and Namkyoo Park

Optical Communication Systems Laboratory, ENG420-015, School of Electrical Engineering and Computer Sciences, Seoul National University, Kwanak, Seoul 151-742, Korea

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We propose a novel framework for the solution of a general fiber Raman amplifier problem by use of a closed integral form of a Raman equation. Treating the given problem as an adiabatic system and taking the Raman process as the perturbation parameter, we can seek the solution along the iteration axis rather than the fiber propagation axis, permitting an orders-of-magnitude increase for the product of convergence speed and spatial resolution in the numerical assessment.

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Because of its wide, flexible gain bandwidth and intrinsically low noise, the fiber Raman amplifier (FRA) has become indispensable in today's high-capacity long-haul optical transmission system.[1] Various FRA modeling methods,[2,3] with different levels of convergence speed and accuracy, have been proposed to produce valuable insights into FRA dynamics and optimum design before experimental implementation. These approaches share the common platform of coupled ordinary differential equations (ODEs) for the Raman equation set that are solved along the length of the fiber propagation axis.

We propose an alternative, highly efficient framework for FRA analysis. By treating Raman gain as a perturbation factor in an adiabatic process, in other words, by adiabatically turning on the Raman process in the fiber, one can obtain the solution of the FRA problem along the iteration axis (for the whole length of fiber) rather than along the fiber propagation axis, facilitating faster convergence speed at a gain accuracy that is equivalent to that achievable with methods based on coupled ODEs. A comparison of the performance shows that our method has more than 60 times the convergence speed of the average power method[3] for the same level of gain accuracy (relative deviation, <0.06 dB).

In treating Raman gain as a perturbation factor we achieved implementation of the algorithm by (1) deriving a recursive relation for the integrals of power inside fiber or, equivalently, the effective length,[4] (2) constructing a matrix formalism for the solution of the given FRA problem, and (3) taking the output power as the final target solution but during the process of optimization considering the effective length an interim target solution for the achievement of faster convergence.

Ignoring the negligible effect of amplified spontaneous emission (ASE) and Rayleigh scattering, we can express the coupled nonlinear Raman process in the fiber as[5]

$$\pm\frac{\mathrm{d}P_i}{\mathrm{d}z} = -\alpha_i P_i + \sum_{j=1}^{M+N} g_{ji} P_j P_i\,, \tag{1}$$

where P_i is the power at ith wavelength, α_i is the attenuation coefficient, g_{ji} is the scaled Raman gain coefficient,[4] M is the pump number, N is the number of signal waves, and the upper (lower) sign indicates a copropagating (counterpropagating) wave. After dividing Eq. (1) by P_i and integrating over z, we get

$$P_i(z) = P_i(0)\exp\left\{\mp\alpha_i z \pm \left[\sum_{j=1}^{M+N} g_{ji}\int_0^z P_j(\zeta)\mathrm{d}\zeta\right]\right\}\cdot \tag{2}$$

With additional integration we get the following integral form of a Raman wave equation:

$$\frac{\int_0^z P_i(\zeta)\mathrm{d}\zeta}{P_i(0)} = \int_0^z \exp\left\{\mp\alpha_i\zeta \pm \left[\sum_{j=1}^{M+N} g_{ji}\int_0^\zeta P_j(\xi)\mathrm{d}\xi\right]\right\}\mathrm{d}\zeta\,. \tag{3}$$

Now, utilizing the definition of $L_{\mathrm{eff}-i}(z) = \int_0^z P_i(z')/P_i(0)\mathrm{d}z'$, we can rewrite Eq. (3) into a closed integral form for the effective length:

$$L_{\mathrm{eff}-i}(z) = \int_0^z \exp\left\{\mp\alpha_i\zeta \pm \left[\sum_{j=1}^{M+N} g_{ji}P_j(0)L_{\mathrm{eff\_}j}(\zeta)\right]\right\}\mathrm{d}\zeta\,. \tag{4}$$

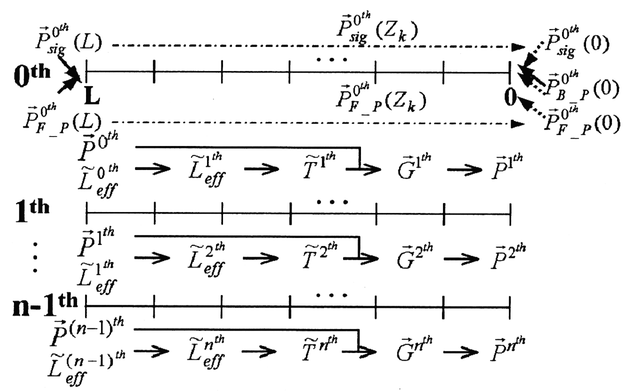

Fig. 1. Flow diagram of the suggested adiabatic iteration algorithm along the iteration axis: $\mathbf{P}_{\mathrm{sig}}^{0\mathrm{th}}$ (0), 0th signal power at $z_k = 0$; $\mathbf{P}_{F-P}^{0\mathrm{th}}$ (0), 0th forward pump power at $z_k = 0$; $\mathbf{P}_{B-P}^{0\mathrm{th}}$ (0), 0th backward pump power at $z_k = 0$.

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To solve Eq. (4) we apply Picard's iteration method to it, taking $L_{\text{eff}-i}$ as the initial, interim target solution. At the nth iteration, Eq. (4) becomes

$$L_{\text{eff\_}i}^{n\text{th}}(z) = \int_0^z \exp\left\{\mp\alpha_i\zeta \pm \left[\sum_{j=1}^{M+N} g_{ji}P_j^{(n-1)\text{th}}(0) \times L_{\text{eff\_}j}^{(n-1)\text{th}}(\zeta)\right]\right\}\mathrm{d}\zeta\,. \quad (5)$$

For the implementation of these equations in the numerical domain we now construct a vector $\mathbf{L}_{\text{eff\_}i}^{n\text{th}}(\mathbf{z}_k)$ to assign the value of the effective length for the ith wavelength at position z_k (the discrete position element covering the whole fiber link in increments of Δz, as shown in Fig. 1) with the vector elements after the nth iteration

$$L_{\text{eff\_}i}^{n\text{th}}(\mathbf{z}_k) = [\ \exp(\mp\alpha_i\mathbf{z}_k \pm \mathbf{g}_{ji}\mathbf{P}_j^{(n-1)\text{th}}(0) \times \tilde{L}_{\text{eff\_All}}^{(n-1)\text{th}}(\mathbf{z}_k)]\tilde{T}_{\text{trig}}\Delta z\,, \quad (6)$$

where

$$\mathbf{z}_k = [0\ \Delta z\ 2\Delta z\ 3\Delta z, \ldots, L - \Delta z\ L],$$

$$L_{\text{eff\_}i}^{n\text{th}}(\mathbf{z}_k) = [L_{\text{eff\_}i}^{n\text{th}}(0) \quad L_{\text{eff\_}i}^{n\text{th}}(\Delta z), \ldots, L_{\text{eff\_}i}^{n\text{th}}(L)],$$

$$\mathbf{g}_{ji}\mathbf{P}_j^{(n-1)\text{th}}(0) = [g_{1i}P_1^{(n-1)\text{th}}(0) \quad g_{2i}P_2^{(n-1)\text{th}}(0), \ldots, g_{M+Ni}P_{M+N}^{(n-1)\text{th}}(0)],$$

$$\tilde{L}_{\text{eff\_All}}^{(n-1)\text{th}}(\mathbf{z}_k) = \begin{bmatrix} \tilde{L}_{\text{eff\_1}}^{(n-1)\text{th}}(\mathbf{z}_k) \\ \tilde{L}_{\text{eff\_2}}^{(n-1)\text{th}}(\mathbf{z}_k) \\ \vdots \\ \tilde{L}_{\text{eff\_}M+N}^{(n-1)\text{th}}(\mathbf{z}_k) \end{bmatrix}$$

$$\tilde{T}_{\text{trig}} = \begin{bmatrix} 0 & 1/2 & 1/2 & 1/2 & \ldots & 1/2 \\ 0 & 1/2 & 1 & 1 & \ldots & 1 \\ 0 & 0 & 1/2 & 1 & \ldots & 1 \\ 0 & 0 & 0 & 1/2 & \ldots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 1/2 \end{bmatrix}.$$

Meanwhile, utilizing the rule of superposition with fractional Raman gains,[5] we can express the Raman gain at a certain wavelength in dB scale as

$$G_i = 10\log(e)\sum_{j=1}^{M+N} g_{ji}L_{\text{eff\_}j}(L)P_j(0)\,. \quad (7)$$

Extending Eq. (7) for every pump or signal wave, we can transform the gain equation into the following matrix form:

$$\mathbf{G} = 10\log(e)\tilde{T} \times \mathbf{P}\,, \quad (8)$$

where

$$\tilde{T} = \begin{bmatrix} g_{11}L_{\text{eff\_1}} & \cdots & g_{M+N1}L_{\text{eff\_}M+N} \\ \vdots & \vdots & \vdots \\ g_{1M+N}L_{\text{eff\_1}} & \cdots & g_{M+NM+N}L_{\text{eff\_}M+N} \end{bmatrix},$$

$$\mathbf{P} = [P_1(0) \quad P_2(0)\ldots P_{M+N}(0)]^T\,.$$

Utilizing this formulation, we can obtain the Raman gains for every combination of pump–signal waves from a single matrix multiplication without relying on the solution of complex, coupled ODEs. To implement the analytical formulation derived above [Eqs. (6) and (8)] in the numerical domain, we take the following steps:

Step I. Set the initial parameter and make the first iteration: Treating $g_{ji} \times P_j(0) \times L_{\text{eff\_}j}$ as a perturbation parameter in the adiabatic process, we can assume that it is equal to zero for the 0th iteration step, to yield a 0th-order initial effective length $\tilde{L}_{\text{eff\_}j}^{0\text{th}} = [1 - \exp(\mp\alpha_j L)]/\alpha_j$. At the same time, to facilitate the application of a transfer matrix for all forward or backward propagating waves we treat the forward propagating waves injected at the input end ($z = L$) as backward propagating waves assigned at the output end[5] ($z = 0$, as illustrated in Fig. 1). To do so, we set $\mathbf{P}_{\text{sig}}^{0\text{th}}(0)$ and $\mathbf{P}_{F-P}^{0\text{th}}(0)$ to be equal to $[\mathbf{P}_{\text{sig}}^{0\text{th}}(L) - |\text{fiber loss}|]$ and $[\mathbf{P}_{F-P}^{0\text{th}}(L) - |\text{fiber loss}|]$, respectively, for the forward propagating waves in the counterpropagating and bidirectional pumping configurations while we use the predetermined initial signal power and forward pump power $\mathbf{P}_{\text{sig}}^{0\text{th}}(0)$ and $\mathbf{P}_{F-P}^{0\text{th}}(0)$, respectively, for codirectional waves.

The effect of this approximation is negligible in the initial iteration because the perturbation $g_{ji} \times P_j(0) \times L_{\text{eff\_}j}$ of the forward propagating signal and the pump is much smaller than that of the backward propagating pump $[\mathbf{P}_{\text{sig}}^{0\text{th}}(0)/\mathbf{P}_{F-P}^{0\text{th}}(0) \ll \mathbf{P}_{B-P}^{0\text{th}}(0)]$.

Step II. Update the effective length, transfer function, gain, and signal–forward pump power: Substituting $\mathbf{P}^{0\text{th}}(0)$ and $\tilde{L}_{\text{eff\_}j}^{0\text{th}}$ into Eq. (6), we can get $\tilde{L}_{\text{eff\_}j}^{1\text{th}}$. With $\mathbf{P}^{0\text{th}}(0)$ and $\tilde{T}^{1\text{th}}$ (calculated from $\tilde{L}_{\text{eff\_}j}^{1\text{th}}$), we can get $\mathbf{G}^{1\text{th}}$ from Eq. (8). To get the first iteration result of target solution $\mathbf{P}^{1\text{th}}(0)$ we update $\mathbf{P}_{\text{sig}}^{1\text{th}}(0)$ as $[\mathbf{P}_{\text{sig}}^{0\text{th}}(L) - |\text{fiber loss}| + \mathbf{G}^{1\text{th}}]$ and $[\mathbf{P}_{F-P}^{1\text{th}}(L) - |\text{fiber loss}| + \mathbf{G}^{1\text{th}}]$, respectively, for the forward propagating waves in the counterpumping–bidirectional pumping scheme, while we use predetermined initial input signal power and forward pump power $\mathbf{P}_{\text{sig}}^{0\text{th}}(0)$ and $\mathbf{P}_{F-P}^{0\text{th}}(0)$, respectively, in the codiretional pumping configuration.

Step III. Use the following reiteration procedure: Repeating the iteration procedure described in Step II with higher-order values of $\tilde{L}_{\text{eff\_}j}$, $\tilde{T}$, $\mathbf{G}$, and $P(0)$, we can obtain the final solution set.

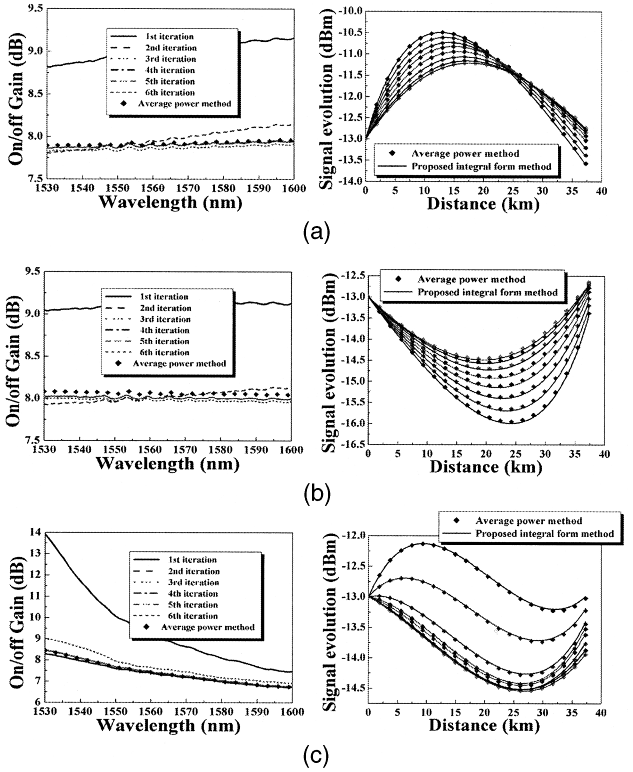

Fig. 2. Raman on–off gain and signal power evolution at a wavelength of 1530–1600 nm (10-nm spacing) for three pumping configurations: (a) codirectional, (b) counterdirectional, and (c) bidirectional.

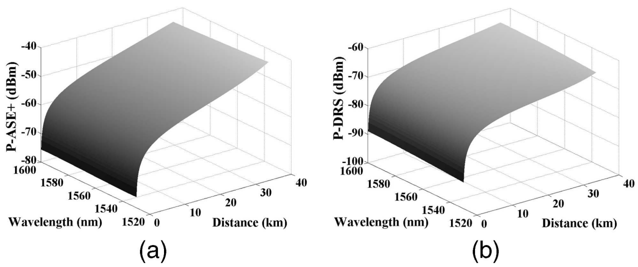

Fig. 3. FRA noise at $T = 300\ \mathrm{K}$ in the counterpumping configuration: (a) forward ASE power, (b) double Rayleigh backscattered (DRS) signal power.

To verify the proposed algorithm we compared our simulation results with those obtained by the average power method[3] (Fig. 2). Seventy-one signals were used at wavelengths of 1530–1600 nm with 1-nm spacing and a signal power of −13 dBm/channel. For the codirectional–counterdirectional pumping scheme, 14 pumps were used (1420–1480 nm with 5-nm spacing, and an additional pump at 1495 nm). For the bidirectional pump, 4 pumps in the forward direction (1420–1435 nm with 5-nm spacing) and 10 pumps in the backward direction (1440–1480 nm with 5-nm spacing and an additional pump at 1495 nm) were used. We used 37 km of dispesion-shifted fiber with a larger Raman gain coefficient that that of single-mode fiber as the test gain medium. As can be seen from Fig. 2, we obtained more than enough convergence accuracy after six iteration steps; the improvement in accuracy from the next order of iteration was less than 0.0001 dB. The relative gain difference between the proposed method and the average power method remained less than 0.06 dB over the whole 71-nm gain spectral range. The required computation time to get the exact Raman gain and signal–pump power distribution with a 100-m step size was less than 0.6 **s** for all the codirectionally; counterdirectionally; and bidirectionally pumping configurations, with a conventional PC (2.0-GHz CPU clock). Obviously, the larger the step size, the shorter the computation time, but at the expense of increased error. The measured convergence speed with the average power analysis method for the same system configuration was much longer (36 s; step size, 1.85 km). Figure 2 also shows the signal's evolution along the fiber. The excellent agreement between the two simulation results shows the stability of the proposed algorithm. Test results of FRAs with high net gain of as much as 10–20 dB (convergence showed dependence on parameters such as loss/gain coefficient, pumping scheme, and signal power) also showed that the stringent design conditions could be met within a subsecond time scale. By using the acquired signal distribution we can also obtain various FRA noises, if necessary, from well-known formulas.[6] Figure 3 shows the various Raman noise components for the counterpumped FRA.

We have proposed and demonstrated a highly efficient framework that replaces the complicated differential Raman equation with a novel closed integral form that can also be used for the design of arbitrarily shaped Raman gain spectra.[4] By updating the effective length along the iteration axis we obtain Raman gain and pump–signal power evolution with orders of magnitude increases in the convergence speed and spatial resolution compared with those obtained with the previous, coupled ODE-based approaches.

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